ANALYSIS OF CURVED BEAMS

Akpo-ezoukumo Excel Douglas

(Tel: +234 7038855240; Email: akpoezou@gmail.com)

Department of Civil Engineering, Niger Delta University, Wilberforce Island, Amassoma, Bayelsa State, Nigeria.

Abstract: **Analysis of curved beam is a vital part of structural analysis as curved beams have found many applications in civil, mechanical and aerospace engineering. This Dissertation developed a computer aided analysis of a "curved beam-element" using Stiffness matrix method. A Microsoft Excel Spreadsheet program was developed and used to analyze the curved beam element. The main assumption is that the curve is made up of finite straight lines therefore stiffness matrix method of analysis is suitable to analyze the curved beam. Abaqus software was used to validate the results obtained from the Excel spreadsheet and the % difference were calculated. The % difference obtained were within acceptable limit so MS Excel program developed can be used to analyze beams. Almost everyone's business or personal computer has Excel installed, which makes it a trustworthy way to transfer information and easy access. For every computer literate that can use the MS Excel Spreadsheet even if you do not know how to use Abaqus, Ansys, FEM or other engineering software which are not always available on all computers, can use the MS Excel spreadsheet to analyze curved beams. The straight beam approximation method is a reliable method to calculate curved beam elements of different section.**

Keywords: **curved, beam, analysis, Ms excel, stiffness matrix.**

1. INTRODUCTION

Curved beams are used in civil, mechanical and aerospace engineering. Beam whose axis is not straight and is curved in the elevation is said to be a curved beam. If the applied loads are along the y direction and the span of the beam is along the x direction, the axis of the beam should have a curvature in the xy plane. On the other hand, if the member is curved on the xz plane with the loading still along the y direction, then it is not a curved beam, as this loading will cause a bending as well as twisting of the section. Thus, a curved beam does not have a curvature in the plan. Arches are examples of curved beams.

Curved beams could be analyzed with different softwares like Ansy, Abaqus, FEM, etc but these softwares are not available on all computers. This dissertation focuses on producing a MS Excel program that could be used to analyze curved beams of different sections and materials. The spreadsheet produced could be used on all computer systems and phones that support Microsoft office files.

2. THE FINITE ELEMENT METHOD

The "finite element method" in structural analysis is a method in which an actual continuous structure is replaced by a mathematical model made up from structural elements of finite size having known material and geometrical properties. The complete structure is then analyzed as an assemblage of these discrete elements, where every such element is treated as a continuous structural member. It is a requirement in this method of structural analysis that the displacements be compatible and the internal forces be in balance at points on the complete structure shared by two or more elements [13]. Such points are called "joints" or "nodes."

The material and geometric properties of each element relate the displacements to the applied generalized forces at each point of the structural element. Thus these properties are called the "force displacement properties" [2] and are related to and depend upon three properties: the stress-strain properties of the material from which the structural element is made, the geometry and sectional properties of the element and the displacement or forces imposed at the boundaries of the structural element.

These force-displacement relationships are usually found either by determining displacements resulting from a known set of forces in equilibrium or determining a set of forces in equilibrium that are required to produce a prescribed configuration of the structural element. These two methods of finding the displacement-force or force-displacement relationships result in two corresponding matrices involving the material and geometric properties of the structural element. The former method results in what is called the "flexibility matrix" while the result of the latter method is the "stiffness matrix the two matrices relate forces and displacements in the following manner:

$$\{v\} = [a]\ (f)$$

$$(F) = [k]\ \{v\}$$

Where {v} = a generalized displacement vector of n components.

{F} = a generalized force vector of n components.

[a] = an nth order square matrix containing the flexibility Coefficients.

[k] = an nth order square matrix containing the stiffness Coefficients.

3. STIFFNESS METHOD

As one of the methods of structural analysis, the **direct stiffness method**, also known as the **matrix stiffness method**, is particularly suited for computer-automated analysis of complex structures including the statically indeterminate type. It is a *matrix* method that makes use of the members' stiffness relations for computing member forces and displacements in structures. The direct stiffness method is the most common implementation of the finite element method (FEM). In applying the method, the system must be modeled as a set of simpler, idealized elements interconnected at the nodes. The material stiffness properties of these elements are then, through matrix mathematics, compiled into a single matrix equation which governs the behaviour of the entire idealized structure. The structure's unknown displacements and forces can then be determined by solving this equation. The direct stiffness method forms the basis for most commercial and free source finite element software.

The direct stiffness method originated in the field of aerospace. Researchers looked at various approaches for analysis of complex airplane frames. These included elasticity theory, energy principles in structural mechanics, flexibility method and matrix stiffness method. It was through analysis of these methods that the direct stiffness method emerged as an efficient method ideally suited for computer implementation.

4. SIMPLE USE OF THE PRE-WRITTEN PROGRAM

The straight beam approximations (stiffness method)

- Insert the individual properties of each beam element on the member properties table (Table 3.1: Input table).
- Insert the nodal force on the Nodes Table (table 3.2 : Input table for external forces on the Nodes)
- To obtain the displacements on the nodes of the structure, we use the formula:
- $D = K^{-1} * F$
- Where:
- D = Displacement
- K = Global Stiffness Matrix
- K^{-1} = Inverse of the global stiffness matrix
- F = Applied forces

5. EXTRACTED EXAMPLES FROM MY DISSERTATION

Question 1: Find the displacements, reactions and final forces of the frame in Fig. Q1 below.

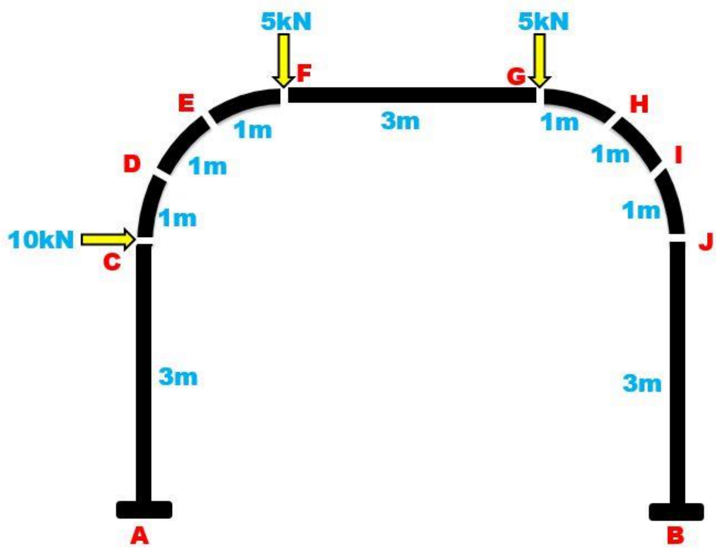

Fig.Q1

Table Q1.1: Properties of all Members in the Frame

MEMBER PROPERTIES					
MEMBER	**L(m)**	**A(m^2)**	**I(m^4)**	**E (KN/m^2)**	**(θ)**
AC	3.000	0.053	0.000233	3.0E+04	90
CD	1.000	0.053	0.000233	3.0E+04	67.5
DE	1.000	0.053	0.000233	3.0E+04	45
EF	1.000	0.053	0.000233	3.0E+04	22.5
FG	2.000	0.053	0.000233	3.0E+04	0
GH	1.000	0.053	0.000233	3.0E+04	337.5
HI	1.000	0.053	0.000233	3.0E+04	315
IJ	1.000	0.053	0.000233	3.0E+04	292.5
BJ	3.000	0.053	0.000233	3.0E+04	270

Table Q1.2: Forces on each Node of the frame

NODES (kN)				
P_{AX}	0		P_{FX}	0
P_{AY}	0		P_{FY}	5
M_A	0		M_F	0
P_{BX}	0		P_{GX}	0
P_{BY}	0		P_{GY}	5
M_B	0		M_G	0
P_{CX}	10		P_{HX}	0
P_{CY}	0		P_{HY}	0
M_C	0		M_H	0
P_{DX}	0		P_{IX}	0
P_{DY}	0		P_{IY}	0
M_D	0		M_I	0
P_{EX}	0		P_{JX}	0
P_{EY}	0		P_{JY}	0
M_E	0		M_J	0

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USING ABAQUS TO SOLVE THE PROBLEM

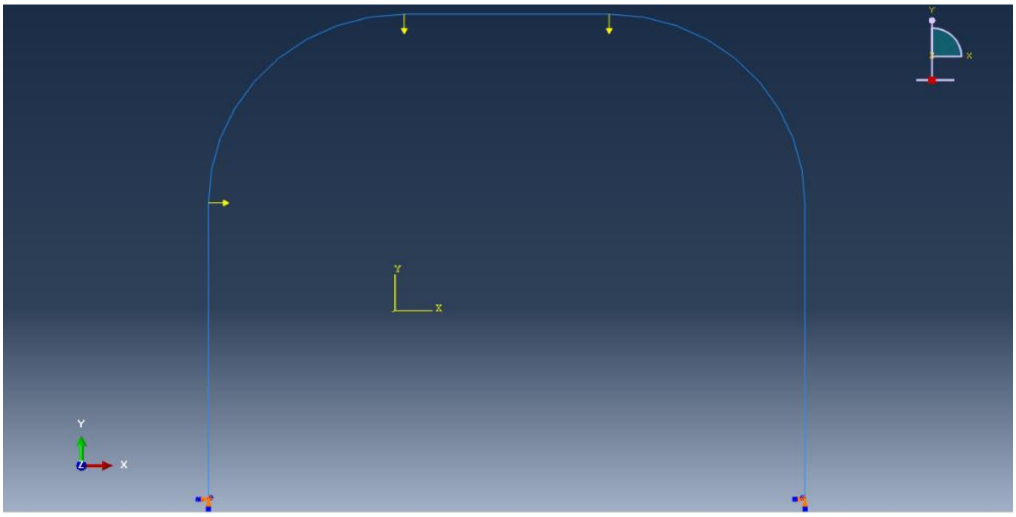

Fig.Q1.1: Loading and Boundary conditions

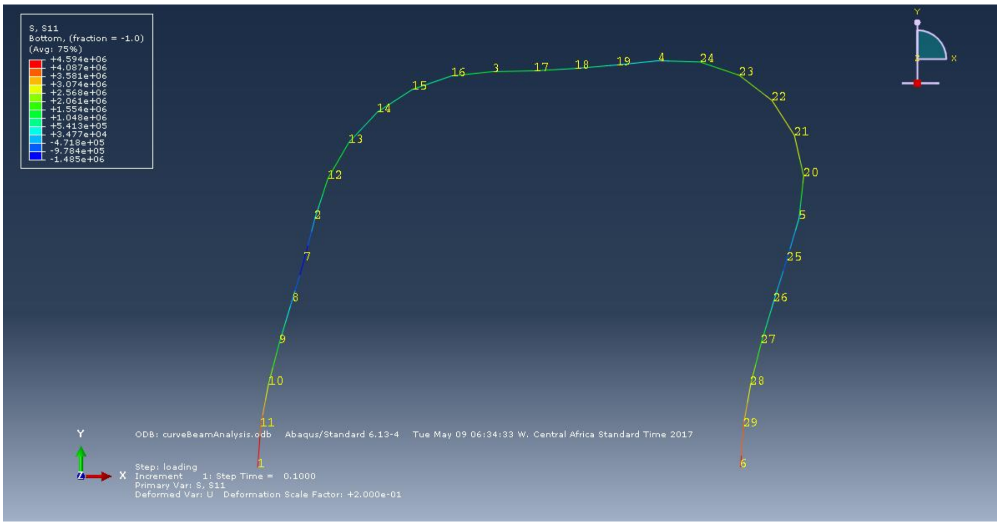

Fig.Q1.10: SS1 plot

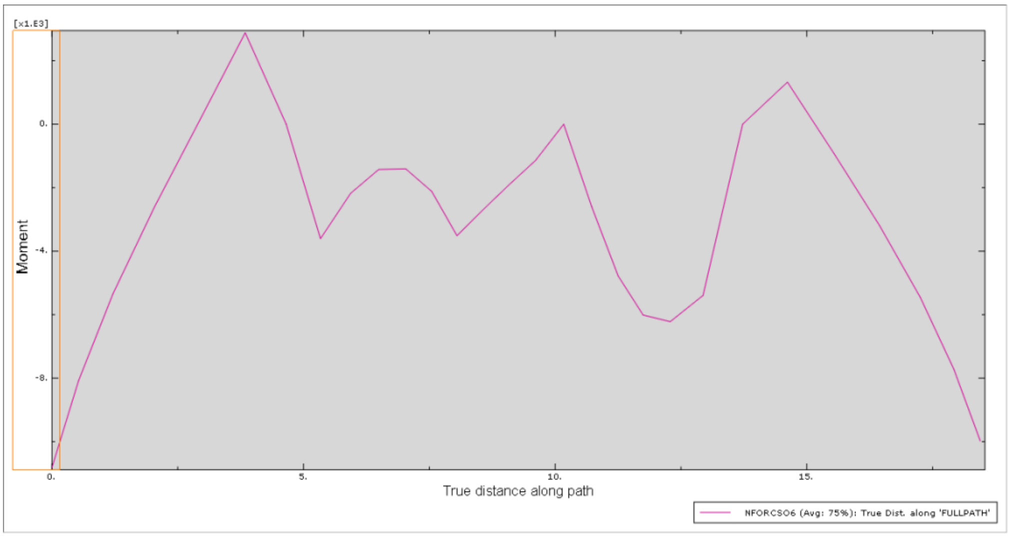

Fig.Q1.11: Bending Moment Diagram (Full Path)

The results obtained from the MS Excel Spreadsheet and Abaqus for Question 1 are displayed in table Q1.3 , Q1.4 and Q1.5 below.

Table Q1.3: Member Displacement Result

	MS EXCEL	ABAQUS	% DIFF
Δ_{AX}	0	5.48E-33	
Δ_{AY}	0	3.42E-33	
Θ_{A}	0	1.08E+04	
Δ_{CX}	4.777	4.69817	1.646081
Δ_{CY}	0.012	1.27E-02	2.592302
Θ_{C}	-1.179	1.26E+00	6.452381
Δ_{CX}	4.777	4.64817	2.692807
Δ_{CY}	0.012	1.27E-02	2.592302
Θ_{C}	-1.179	-1.26E+00	6.89743
Δ_{DX}	5.191	5.048	2.752894
Δ_{DY}	-0.155	-0.1671	7.73694
Θ_{D}	0.211	0.18	14.73235
Δ_{DX}	5.191	5.08	2.136431
Δ_{DY}	-0.155	-0.1494	-3.81526
Θ_{D}	0.211	0.22815	7.473154
Δ_{EX}	4.717	5.0128	5.896904
Δ_{EY}	0.322	0.299	7.229289
Θ_{E}	1.012	0.98159	3.014524
Δ_{EX}	4.717	5.043	6.46044
Δ_{EY}	0.322	0.296	8.160099
Θ_{E}	1.012	0.98132	3.041201
Δ_{FX}	4.305	4.60066	6.417775
Δ_{FY}	1.32	1.20E+00	9.278569
Θ_{F}	0.995	1.015	1.970443
Δ_{FX}	4.305	4.460066	3.467796
Δ_{FY}	1.32	1.27E+00	3.637466
Θ_{F}	0.995	1.015	1.970443
Δ_{GX}	4.305	4.59496	6.319097
Δ_{GY}	2.07	1.89E+00	8.730734
Θ_{G}	-0.395	-0.37824	4.48393
Δ_{GX}	4.305	4.59496	6.319097
Δ_{GY}	2.07	1.89E+00	8.730734
Θ_{G}	-0.395	-0.37824	4.48393
Δ_{HX}	4.009	3.89496	2.837329
Δ_{HY}	1.354	1.267	6.439226
Θ_{H}	-1.072	-0.99722	7.47879
Δ_{HX}	4.009	4.18	4.098086
Δ_{HY}	1.354	1.358	0.279823
Θ_{H}	-1.072	-1.1599	8.21982
Δ_{IX}	3.147	3.143	0.139798
Δ_{IY}	0.491	0.485	1.262215
Θ_{I}	-1.298	-1.395	7.47304
Δ_{IX}	3.147	2.92	7.225011
Δ_{IY}	0.491	0.502	2.151394
Θ_{I}	-1.298	-1.1955	8.57382
Δ_{JX}	1.982	2.152196	7.912662

Δ_{JY}	0.006	5.91E-03	9.04
Θ_J	-1.181	-1.20986045	2.42638
Δ_{BX}	0	4.52E-33	
Δ_{BY}	0	6.58E-33	
Θ_B	0	0.000056	
Δ_{JX}	1.982	2.152196	7.912662
Δ_{JY}	0.006	5.92E-03	8.861538
Θ_J	-1.181	-1.20913	2.36454

Table Q1.4: Reactions on the Nodes of the Frame

	MS EXCEL	ABAQUS	% DIFF
P_{ACX}	-9.3473	-10.0048	7.03412
P_{ACY}	-6.5764	-6.6157	0.59759
M_{AC}	16.76733	16.456	1.85659
P_{CAX}	9.347299	9.256	0.976753
P_{CAY}	6.576397	6.496	1.222553
M_{CA}	11.27457	10.946	2.914516
P_{CDX}	0.652701	0.6558	0.472705
P_{CDY}	-6.5764	-6.998	6.4108
M_{CD}	-11.2746	-11.44	1.46701
P_{DCX}	-0.6527	-0.64511	1.17654
P_{DCY}	6.576397	7.01585	6.263674
M_{DC}	8.154872	7.887	3.285141
P_{DEX}	0.652701	0.6258	4.121342
P_{DEY}	-6.5764	-6.995	6.36518
M_{DE}	-8.15487	-7.9582	2.47166
P_{EDX}	-0.6527	-0.696	6.63398
P_{EDY}	6.576397	7.0246	6.380434
M_{ED}	3.043128	2.859	6.049752
P_{EFX}	0.652701	0.696	6.221264
P_{EFY}	-6.5764	-6.992	6.36518
M_{EF}	-3.04313	-2.852	6.70056
P_{FEX}	-0.6527	-0.6496	0.47722
P_{FEY}	6.576397	6.47857	1.487592
M_{FE}	-3.28245	-3.456	5.28881
P_{FGX}	0.652701	0.5986	8.288647
P_{FGY}	-1.5764	-1.617	2.57549
M_{FG}	3.282449	3.198	2.571289
P_{GFX}	-0.6527	-0.6791	4.04474
P_{GFY}	1.576397	1.61686	2.502381
M_{GF}	-6.43524	-5.9576	8.01665
P_{GHX}	0.652701	0.646	1.026505
P_{GHY}	3.423603	3.591	4.661654
M_{GH}	6.435242	6.572	2.081558
P_{HGX}	-0.6527	-0.644	1.35093
P_{HGY}	-3.4236	-3.582	4.62671
M_{HG}	-3.02247	-2.96	2.11149
P_{HIX}	0.652701	0.67	2.58209
P_{HIY}	3.423603	3.72	7.967742
M_{HI}	3.022468	2.81	7.030604
P_{IHX}	-0.6527	-0.68	4.18263

P_{IHY}	-3.4236	-3.49	1.93948
M_{IH}	-0.14009	-0.1469	4.85368
P_{IJX}	0.652701	0.6397	1.991727
P_{IJY}	3.423603	3.554	3.669105
M_{IJ}	0.140086	0.1385	1.142041
P_{JIX}	-0.6527	-0.71	8.77892
P_{JIY}	-3.4236	-3.387	1.0806
M_{JI}	1.773087	1.799	1.439689
P_{BJX}	-11.6613	-12.17	4.36319
P_{BJY}	-3.4236	-3.368	1.65083
M_{BJ}	-14.7397	-15.02	1.90167
P_{JBX}	11.66125	12.07	3.38691
P_{JBY}	3.423603	3.368	1.624021
M_{JB}	-20.244	-19.702	2.75099

Table Q1.5: Final Member Forces of the Frame

	MS EXCEL	**ABAQUS**	**% DIFF**
P_{ACX}	-6.576	-6.59	0.2068
P_{ACY}	9.347	8.918	4.59277
M_{AC}	16.77		
P_{CAX}	6.576	6.59	0.206373
P_{CAY}	-9.347	-8.918	4.81386
M_{CA}	11.27		
P_{CDX}	-5.826	-6.087	4.47992
P_{CDY}	-3.12	-2.95789	5.47045
M_{CD}	-11.27		
P_{DCX}	5.826	6.0287	3.362251
P_{DCY}	3.12	3.0789	1.307818
M_{DC}	8.155		
P_{DEX}	-4.189	-3.9814	5.20671
P_{DEY}	-5.112	-5.245	2.60774
M_{DE}	-8.155		
P_{EDX}	4.189	4.278	2.087424
P_{EDY}	5.112	4.945	3.261146
M_{ED}	3.043		
P_{EFX}	-1.914	-1.98	3.46449
P_{EFY}	-6.326	-6.48	2.44088
M_{EF}	-3.043		
P_{FEX}	1.914	2.0287	5.668655
P_{FEY}	6.326	6.178	2.333375
M_{FE}	-3.282		
P_{FGX}	0.653	0.641675	1.689137
P_{FGY}	-1.576	-1.54849	1.8024
M_{FG}	3.282		
P_{GFX}	-0.653	-0.621675	4.99055
P_{GFY}	1.576	1.4849	5.804364
M_{GF}	-6.435		
P_{GHX}	-0.707	-0.7568	7.02871
P_{GHY}	3.413	3.68	7.26087
M_{GH}	6.435		

P_{HGX}	0.707	0.71568	1.19886
P_{HGY}	-3.413	-3.368	1.33017
M_{HG}	-3.022		
P_{HIX}	-1.959	-1.84	6.4837
P_{HIY}	2.882	2.7921	3.132806
M_{HI}	3.022		
P_{IHX}	1.959	2.0754	5.594102
P_{IHY}	-2.882	-3.021	-4.80849
M_{IH}	-0.14		
P_{IJX}	-2.913	-3.1047	6.57353
P_{IJY}	1.913	1.891	1.16036
M_{IJ}	0.14		
P_{JIX}	2.913	3.012	3.280212
P_{JIY}	-1.913	-2.0172	5.43592
M_{JI}	1.773		
P_{BJX}	3.424	3.25	5.070686
P_{BJY}	-11.66	-11.024	5.78012
M_{BJ}	-14.74		
P_{JBX}	-3.424	-3.34	2.50299
P_{JBY}	11.66	11.131	4.546702
M_{JB}	-20.24		

The percentage difference between the results obtained from the abaqus and the MS Excel calculation sheet is within acceptable limit. Therefore, this proves that the MS Excel calculation sheet is a good one for calculating curved beams.

To further validate the results from the MS Excel program we solved a problem solved as an example in page 412 of STRUCTURAL ANALYSIS USING CLASSICAL AND MATRIX METHOD, written by JAMES K. NELSON AND JACK C. MCCORMAC THIRD EDITION and the results are compared below.

Question 5: Find the final forces of the frame in Fig. Q5 below.

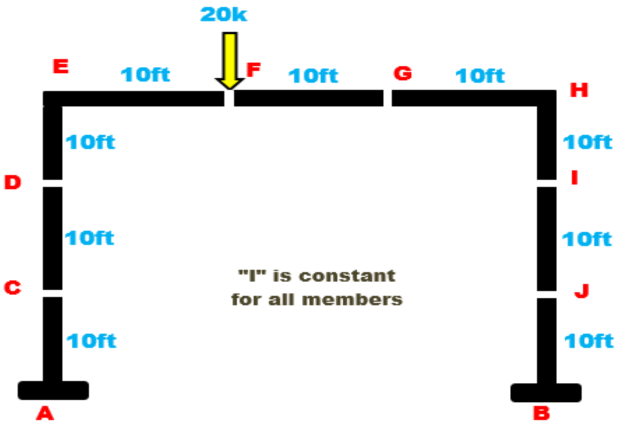

Fig.Q5

The results obtained from the MS Excel and that of the textbook are shown below in table Q5.2 together with the % difference.

Table Q 5.1: Properties of all Members in the Frame

MEMBER PROPERTIES					
MEMBER	L(ft)	A(m^2)	I(m^4)	E (KN/m^2)	(θ)
AC	10	0.053	0.0002332	3.0E+04	90
CD	10	0.053	0.0002332	3.0E+04	90

DE	10	0.053	0.0002332	3.0E+04	90
EF	10	0.053	0.0002332	3.0E+04	0
FG	10	0.053	0.0002332	3.0E+04	0
GH	10	0.053	0.0002332	3.0E+04	0
HI	10	0.053	0.0002332	3.0E+04	270
IJ	10	0.053	0.0002332	3.0E+04	270
BJ	10	0.053	0.0002332	3.0E+04	270

Table Q5.2: Final Member Forces of the Frame

	Textbook (kips), (k-ft)	MS Excel (kips), (k-ft)	% Diff
P_{ACX}	-13.93	-13.53	2.95639
P_{ACY}	2.67	2.215	17.0412
M_{AC}	26.7	19.1	28.46442
P_{CAX}	13.93	13.53	2.8715
P_{CAY}	-2.67	-2.215	20.5418
M_{CA}	2.7	3.047	11.38825
P_{EDX}	13.93	13.53	2.8715
P_{EDY}	-2.67	-2.215	20.5418
M_{ED}	53.3	47.34	11.18199
P_{EFX}	-2.67	-2.215	20.5418
P_{EFY}	13.93	-13.53	2.95639
M_{EF}	-53.3	-47.34	12.5898
P_{GHX}	-1.78	-2.215	24.4382
P_{GHY}	6.07	6.468	6.15337
M_{GH}	35.5	23.29	34.39437
P_{HGX}	1.78	2.215	19.63883
P_{HGY}	-6.07	-6.468	6.55684
M_{HG}	35.5	41.4	14.25121
P_{HIX}	-6.07	-6.468	6.55684
P_{HIY}	-1.78	-2.215	24.4382
M_{HI}	-35.5	-41.4	16.6197
P_{BJX}	6.07	6.468	6.15337
P_{BJY}	17.8	18.98	6.217071
M_{BJ}	78.9	80.94	2.520385
P_{JBX}	6.07	-6.468	6.55684
P_{JBY}	17.8	-18.98	6.62921
M_{JB}	106.2	108.9	2.479339

All results obtained are compared and the % difference between the MS excel and the Textbook are within the acceptable range.

Several examples were considered using other softwares like Matlab, Abaqus, Winkler's method and the MS Excel. The results obtained and the percentage differences were calculated and submitted to the department of Civil Engineering in Niger Delta University, Amassoma, Bayelsa State and the results were approved by Dr. Solomon T. Orumu, Professor Adewumi K. Ife and the department for my masters degree dissertation.

6. CONCLUSION

From the results shown in the above example, it can be said that the MS Excel spreadsheet develop using the stiffness matrix method yields an approximate result to the Abaqus software results. The developed MS Excel Spreadsheet

Program can be used to analyze curved beams and for every computer literate that can use the Ms Excel Spreadsheet even if you do not know how to use Abaqus, Ansy, FEM, etc, which are not available on all computers.

Based on the results obtained from the example above, the stiffness matrix method of curved beam analysis gives an approximate result to the exact method. The following recommendations are made:

1. The method is an approximate method
2. An increase in the number of straight lines would yield more accurate results.
3. The longer the length of the curve, more straight lines are required.
4. A more familiar software (MS-Excel Spreadsheet) can be used to solve analyze curved beam elements.

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